2009B #5

(a) STATE: $\mu \rightarrow$ true mean amount of juice dispensed into bottles (ounces)

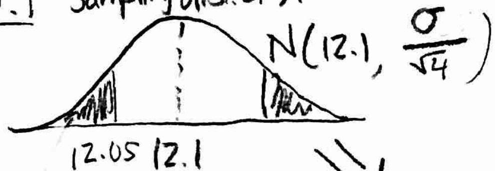
$H_0: \mu = 12.1$ $\quad \alpha = 0.05$ $\quad \bar{x} = 12.05$

$H_a: \mu \neq 12.1$ $\quad s_x = 0.085$

PLAN: one sample t-test for μ

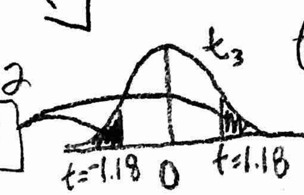
- Random: "four randomly selected bottles" $\rightarrow$ so we can generalize to all bottles
  - 10% condition $4 \leq \frac{1}{10}$(all bottes) $\rightarrow$ so sampling without replacement is OK.
- Normal: the population distribution is approximately normal $\rightarrow$ so the sampling dist. is approximately normal (we can use t-dist)

DO: Sampling dist. of $\bar{x}$



Test statistic $= \dfrac{\text{statistic} - \text{parameter}}{\text{SD statistic}}$

$t = \dfrac{\bar{x} - \mu}{s_x/\sqrt{n}} = \dfrac{12.05 - 12.1}{0.085/\sqrt{4}} = -1.18$



P-value = 2×0.162

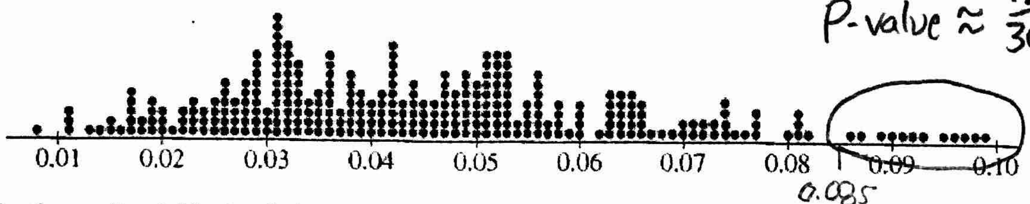
$\boxed{\text{P-value} = 0.324}$ $\quad \boxed{t = -1.18}$ $\quad df = 3$

CONCLUDE:

Assuming H_0 is true ($\mu = 12.1$), there is a 0.324 probability of getting an $\bar{x}$ value that is 0.05 or more away from the mean, purely by chance. This provides weak evidence against H_0 and is NOT statistically significant ($0.324 > 0.05$). Therefore we fail to reject H_0 and do not have convincing evidence that the mean amount of juice dispensed is different from 12.1 ounces.

(b) To determine whether this sample of four bottles provides convincing evidence that the standard deviation of the amount of juice dispensed is greater than 0.05 ounce, a simulation study was performed. In the simulation study, 300 samples, each of size 4, were randomly generated from a normal population with a mean of 12.1 and a standard deviation of 0.05. The sample standard deviation was computed for each of the 300 samples. The dotplot below displays the values of the sample standard deviations.

P-value $\approx \frac{12}{300} = 0.04$



Use the results of this simulation study to explain why you think the sample provides or does not provide evidence that the standard deviation of the juice dispensed exceeds 0.05 fluid ounce.

Assuming the true standard deviation is 0.05, there is a 0.04 probability of getting a sample standard deviation of 0.085 or greater purely by chance. Because $0.04 < 0.05$ we have convincing evidence that the standard deviation of the juice dispensed exceeds 0.5.