Statistics and the Redesigned SAT

Pasadena Public Schools

February 17, 2020

Lindsey Gallas
East Kentwood High School
Kentwood, Michigan
StatsMedic.com
lindsey@statsmedic.com

STATS MEDIC

Agenda:

1. Introductions
2. Statistics and the Redesigned SAT
3. Four Unit Statistics Course

   Lesson Modeling

   Unit 1: Probability

   - Who will get the Last Banana?
   - Can you Taco Tongue or Evil Eyebrow?

     Sample SAT Problems

   Reflection on Lesson Experiences, Q & A time

   Unit 2: One Variable Data

   - How many hours do you watch Netflix? Day 2

     Sample SAT Problems

   Unit 3: Two Variable Data

   - How much candy can you grab?
   - How much candy can LeBron grab?

     Sample SAT Problems

   Unit 4: Statistical Thinking and Inference

   - Does Beyoncé write her own lyrics?

     Sample SAT Problems

4. Reflections and Closure

Goals for today:

- Participants will become familiar with the statistics and probability standards that are included on the SAT.
- Participants will be exposed to the types of questions that are on the SAT.
- Participants will actively participate in activities that develop deeper understandings of statistical concepts.
- Participants will become familiar with the Experience First, Formalize Later lesson design.
- Participants will be prepared to teach four units (Probability, One Variable Data, Two Variable Data, and Statistical Thinking and Inference) of statistical curriculum in the core mathematics classes.

Statistics Unit
Schedule and Learning Targets

Unit 1: Probability		
	Topic	Learning Targets
Lesson 1.1	Probability Models and Rules Activity: Who will win the Last Banana?	Understand that probabilities describe the results of a long run. Learn and apply basic rules of probability including: Probability models add up to 1 $P(A^c) = 1 - P(A)$ Addition rule for mutually exclusive events: $P(A \text{ or } B) = P(A) + P(B)$ $P(\text{At least } 1) = 1 - P(\text{None})$ Use proper notation to describe the probability of an event.
Lesson 1.2	Probability using Two-Way Tables Activity: Can you Taco Tongue and Evil Eyebrow?	Decide if two events are mutually exclusive. Apply the general addition rule for any two events: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. Use two-way tables to find probabilities. Use two-way tables to find conditional probabilities.
Lesson 1.3	Probability using Tree Diagrams Activity: Can you get Kings or Aces?	Decide if two events are independent. Apply the general multiplication rule for any two events: $P(A \text{ or } B) = P(A) \times P(B\|A)$. Use tree diagrams to find the probability of more than one event occurring.

Unit 2: One Variable Data		
	Topic	Learning Targets
Lesson 2.1	Categorical Data and Displays Activity: What's your favorite social media platform?	Identify categorical and quantitative variables. Organize data in frequency and relative frequency tables. Interpret bar charts and pie charts.
Lesson 2.2	Measures of Center for Quantitative Data Activity: How many hours do you watch Netflix?	Calculate a mean and median from a set of values and from a graph of a distribution. Understand the effects of outliers on a mean or median. Decide which measure of center is most appropriate for a data set.
Lesson 2.3	Measures of Spread for Quantitative Data	Calculate a range and standard deviation from a data set. Understand the effect of outliers on range and standard deviation. Use the graph of a set of data to make decisions about standard deviation.
Lesson 2.4	Describing Distributions	Describe the shape, center, spread and outliers of a distribution of quantitative data. Describe how shape relates to the locations of the mean and median. Decide which measure of spread is most appropriate for a data set.

Unit 3: Two Variable Data		
	Topic	Learning Targets
Lesson 3.1	Scatterplots and Line of Best Fit Activity: How much candy can you grab?	Identify explanatory and response variables. Create and describe a scatterplot for two variable data. Interpret the slope and *y*-intercept of a line of best fit.
Lesson 3.2	Predictions and Residuals Activity: How much candy can LeBron grab?	Use a line of best fit to make predictions. Calculate and interpret a residual using linear models.
Lesson 3.3	Models for Non-linear Data Activity: How many iPhones will sell?	Describe non-linear models. Calculate and interpret a residual using non-linear models.

Unit 4: Sampling and Inference		
	Topic	Learning Targets
Lesson 4.1	Random Sampling Activity: Does Beyoncé write her own lyrics?	Understand the difference between a population and a sample. Understand that each sample produces a different estimate. With random samples, describe which population the results can be extended to.
Lesson 4.2	Margin of Error Activity: Who will win the Iowa primary?	Use sample mean and sample proportion to estimate population mean and population proportion. Interpret and utilize, but do not calculate, margin of error. Understand that a larger sample size leads to a smaller margin of error.
Lesson 4.3	Observational Study vs. Experiment Does SAT prep course improve SAT? Day 1	Understand why random assignment provides evidence for a causal relationship. Given a description of a study with or without random assignment, determine whether there is evidence for a causal relationship. Understand the difference between correlation and causation.
Lesson 4.4	Random Sample vs. Random Assignment Activity: Does SAT prep course improve SAT? Day 2	Understand why a result can be extended only to the population from which the sample was selected. Given a description of a study with or without random assignment, determine whether there is evidence for a causal relationship.

Name: _____ Hour:_____ Date: _____

Who will win the Last Banana?

Suppose that you're on a desert island playing dice with another castaway. The winner's prize will be the last banana. Here are the rules of the game:

- Each player rolls a die
- If the largest value shown is a 1, 2, 3, or 4, then Player 1 wins
- If the largest value shown is a 5 or 6 then Player 2 wins

1. Who do you think has advantage in this game: Player 1, Player 2, or neither? Make your **best guess** and explain your choice.

2. Play the game 20 times with your partner and record the winner of each game by tallying in the table below.

Player	**1**	**2**
Tally/Count of Wins		
Percentage of Wins		

   a. How many times did Player 1 win? _____ Write this as a proportion. _____

   b. How many times did Player 2 win? _____ Write this as a proportion. _____

3. Who won more often? Maybe this was only true for your group. Let's see how the rest of the class did. Write the number of wins for Player 1 in the table on the board.

   a. Find the total proportion of wins for Player 1 for the whole class.

   b. Find the total proportion of wins for Player 2 for the whole class.

4. To determine the true probability of Player 1 winning, we should list out all possible rolls that we could get. Complete the table below to show all possible rolls.

   a. Use your table to find the probability of Player 1 winning.

   b. Which was closer to the probability you found in #4a, your group data or the whole class data? Why do you think that is?

	1	2	3	4	5	6
1	1,1					
2						
3						
4						
5						
6						

Name: _____ Hour:_____ Date: _____

Lesson 1.1– Probability Models and Rules

Important ideas:

Check Your Understanding:

What is your favorite color Skittle? Based on a very large sample, here is the distribution of color for Skittles that come in fun-sized bags.

Color	Green	Purple	Orange	Red	Yellow
Probability	0.171	0.214	0.204	???	0.214

1. Explain what the probability of 0.214 for purple means in this setting.

2. Find the probability for red.

3. What is the probability that a randomly selected Skittle is green or purple?

4. What is the probability that a randomly selected Skittle is not yellow?

Name: _____ Hour: _____ Date: _____

Can You Taco Tongue and Evil Eyebrow?

Some people believe that the ability to taco tongue and evil eyebrow is something that you are born with. Is this true? Are the two abilities somehow related?

1. Collect class data to fill in the following two-way table.

		Evil Eyebrow			
		No	Sort of	Yes	Totals
Taco Tongue	Yes				
	No				
	Totals				

2. Suppose that we randomly choose a student from class. Find the following probabilities.

   P(No Evil Eyebrow) =

   P(Sort of Evil Eyebrow) =

   P(No Evil Eyebrow OR Sort of Evil Eyebrow) =

3. Suppose that we randomly choose a student from class. Find the following probabilities.

   P(Yes Taco Tongue) =

   P(Yes Evil Eyebrow) =

   P(Yes Taco Tongue OR Yes Evil Eyebrow) =

4. Suppose that we randomly choose a student from class. Find the following probabilities.

   P(Yes Evil Eyebrow) =

   P(Yes Evil Eyebrow, given the person is Yes Taco Tongue) =

   P(Yes Evil Eyebrow, given the person is No Taco Tongue) =

5. What do your results from #4 tell you about the ability to Taco Tongue and the ability to Evil Eyebrow?

Name: _____ Hour: _____ Date: _____

Lesson 1.2: Probability using Two-Way Tables

Important Ideas:

Check Your Understanding:

Students at a small high school in Michigan were asked, "Which subject area do you prefer: Math or English?" For each student, their grade level and response were recorded.

	Freshman	Sophomores	Juniors	Seniors	Total
Math	20	25	34	40	119
English	44	35	31	21	131
Total	64	60	65	61	250

(a) If a student is randomly selected, what is the probability that are a Senior who prefers Math?

(b) If a student who prefers Math is randomly chosen, what is the probability they are a Freshman?

(c) If a Freshman student is randomly chosen, what is the probability they prefer Math?

2. Suppose a student from this high school is randomly selected, find the following:

(a) P(Math and Freshman)

(b) P(English, given they are a Junior or Senior)

(c) P(Sophomore or Senior)

(d) P(Sophomore or Math)

Name: _____ Hour: _____ Date: _____

How many hours do you watch Netflix? Day 2

In the last lesson we talked about how many hours of Netflix students watch on average. A group of 6 students reported the number of hours watched. The mean of the number of hours watched for the group was 5 hours.

1. If the mean number of hours watched was 5 hours, does that mean each of the 6 students **must** have each watched 5 hours? Why?

2. With your partner, make a possible set of 6 values that have a mean of 5 hours.

3. Compare your list with another pair of students. Find the range of each list. Who's list is more spread out?

   Your range: Their range:

4. The actual set of values for these 6 students was 2, 3, 5, 6, 6, and 8. Calculate the range.

5. Finding range is helpful but it does not tell us how spread out the data is between the minimum and maximum. How can we find the ***average distance of the values from the mean***?

   a. Find the mean of the data.

   b. Find the distance of each piece of data from the mean.

   c. Square the differences to make them positive.

   d. Find the average.

   e. Square root to undo the squares.

6. Interpret the standard deviation.

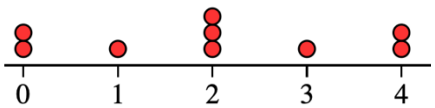
Name: _____ Hour: _____ Date: _____

Lesson 2.3 – Measures of Spread

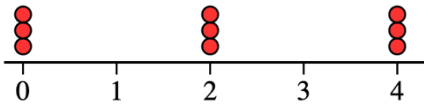
Important ideas from the text:

Check Your Understanding

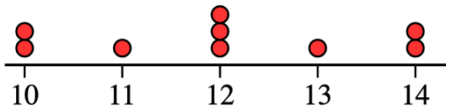
Use the dotplots for the given data sets to make comparisons. Explain.



Data Set A



Data Set B



Data Set C

1. Fill in the blanks with <, >, or = to create a true statement.

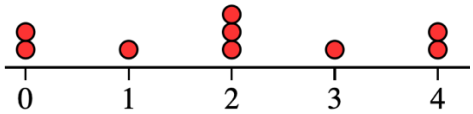
   a. Mean of Data Set A _____ Mean of Data Set C

   b. Standard Deviation of Data Set A _____ Standard Deviation of Data Set B

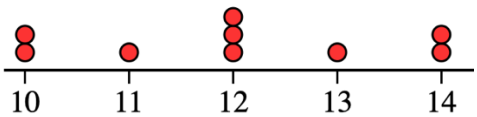
   c. Standard Deviation of Data Set A _____ Standard Deviation of Data Set C

   d. Range of Data Set B _____ Range of Data Set C

2. If we add a value at 12, how will that affect the standard deviation of this data set. Explain.



3. If we add a value at 12, how will that affect the standard deviation of this data set. Explain.



Name: _____ Hour: _____

How much candy can you grab?

Can students with a larger handspan grab more candy than those with smaller handspans? Today we will investigate this question.

1. Measure the span of your dominant hand to the nearest half centimeter (cm). Handspan is the distance from the tip of the thumb to the tip of the pinkie finger on your fully stretched-out hand. Handspan = _____ cm

2. Use the same hand to grab as many candies as possible from the container. You must grab the candies with your fingers pointing down (no scooping!) and hold the candies for 2 seconds before counting them. After counting, put the candy back into the container. Record your data in the spreadsheet.

3. Use the applet at www.statsmedic.com/applets (2 Quantitative Variables) to make a scatterplot. Sketch below.

4. Describe the relationship displayed in the scatterplot.

5. Use the applet to find the line of best fit. Record it below.

6. What is the slope of the line? Interpret the slope in context.

7. What is the y-intercept of the line? Interpret the y-intercept in context.

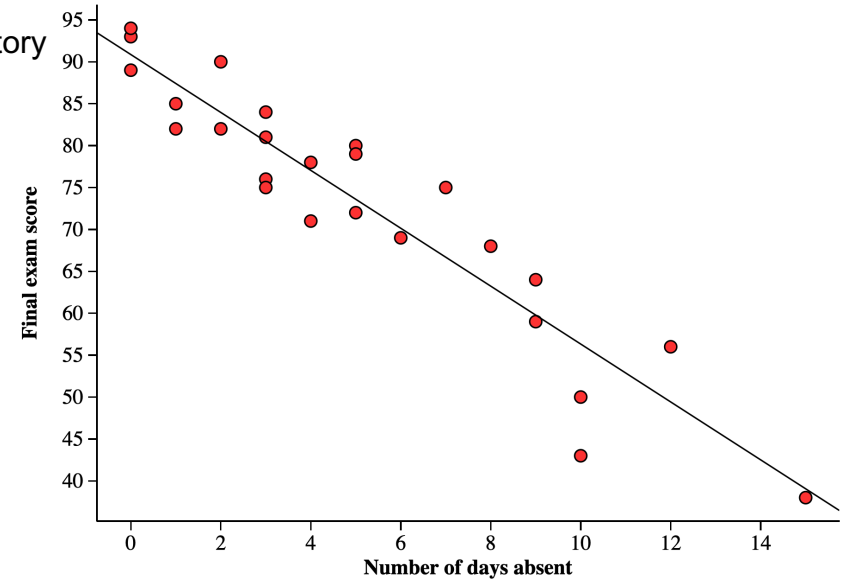
Name: _____ Hour: _____

Lesson 3.1– Scatterplots and Line of Best Fit

Important ideas:

Check Your Understanding

At the end of a semester, a math teacher wonders if student attendance has an impact on final exam scores. Here is a scatterplot that shows the number of days absent and final exam score for a class or 25 students.



1. Identify which variable is the explanatory and which is the response.

2. Describe the relationship.

3. The line of best fit is $\hat{y} = 90.9 - 3.5x$, where x = number of days absent and y = final exam score.

(a) Interpret the slope of the line of best fit.

(b) Interpret the y-intercept of the line of best fit.

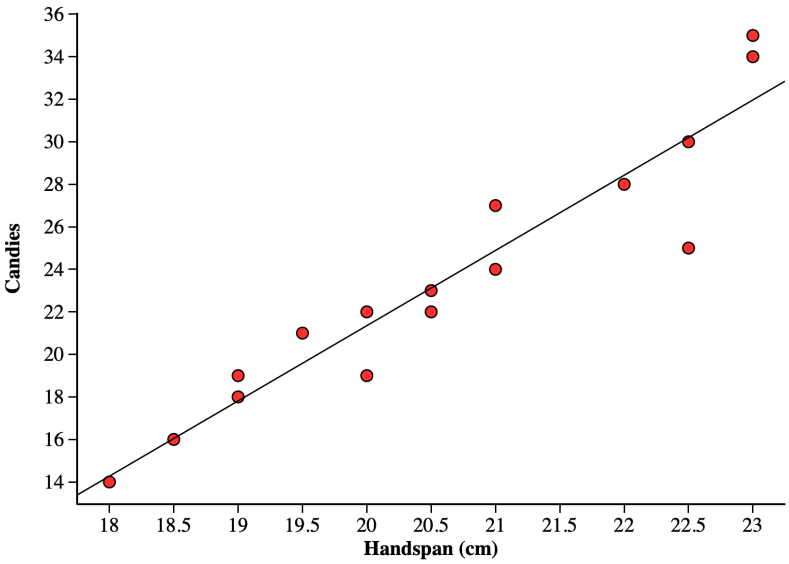
Name: _____ Hour: _____

How much candy can LeBron grab?

LeBron James has four NBA MVP Awards, three NBA Finals MVP Awards, and two Olympic Gold Medals. But how many candies can he grab?

1. One of the Algebra classes collected the following data:

Handspan (x)	18	18.5	19	19	19.5	20	20	20.5	20.5	21	21	22	22.5	22.5	23	23
Candies (y)	14	16	19	18	21	19	22	23	22	24	27	28	25	30	34	35



2. Use the applet to find the line of best fit. Record it below.

3. LeBron James has a handspan of 23.5 cm. Use the equation of the line to predict how many candies Lebron can grab. Show your work?

4. When LeBron visited East Kentwood High School, he attempted the candy grab and was able to grab 38 candies.

   a. Add this point to the scatterplot
   b. Was this value higher or lower than what you predicted?
   c. By how much? Show your work.

5. Your teacher will now measure their handspan. Handspan = _____

6. Predict how many candies your teacher can grab. Show your work.

7. Your teacher will now attempt the candy grab. Number of candies = _____

8. Using the data for your teacher:

   a. Add this point to the scatterplot
   b. Was this value higher or lower than what you predicted?
   c. By how much? Show your work.

STATS MEDIC

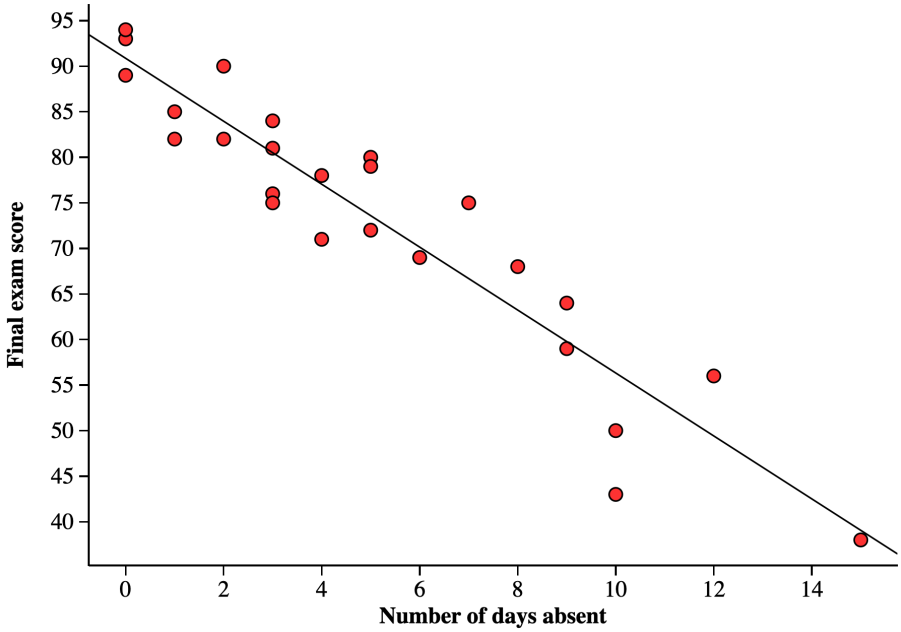
Name: _____ Hour: _____

Lesson 3.2 – Predictions and Residuals

Important ideas:

Check Your Understanding

At the end of a semester, a math teacher wonders if student attendance has an impact on final exam scores. Here is a scatterplot that shows the number of days absent and final exam score for a class or 25 students. The line of best fit is $\hat{y} = 90.9 - 3.5x$, where *x* = number of days absent and *y* = final exam score.



1. Find the predicted final exam score for a student 7 absences. Show your work.

2. According to the scatterplot, what did the student with 7 absences actually score on the final exam?

3. Calculate the residual for the student with 7 absences. Interpret the residual.

4. For how many students was the prediction greater than the actual?

Name: _____ Hour: _____ Date: _____

Lesson 4.1: Does Beyoncé write her own lyrics?

Bey BEYONCÉ CRAZY IN LOVE

1. Quickly circle a random sample of 5 words. Write them below. How many letters are in each word?

2. What is the average word length of your sample of 5 words? _____.

3. What was your partner's sample average? Was it the same as yours? Why?

4. Do you think your sample average is exactly the same as the true average word length of all of the words in Crazy in Love? Why?

5. Write your **sample mean** on a sticker dot and add it to the class dotplot. What do you think the true mean word length is? Why?

STOP

6. Did you really choose your words **randomly**? Find a new sample of 5 words using a random number generator. Put your new sample mean a sticker dot and add it to the new class dotplot. Sketch the class dotplot below.

7. How is the dotplot from #6 different than the dotplot for #5? Which do you think is a better estimator of the true mean word length of Crazy in Love?

8. What do you think the true mean word length is for Crazy in Love?

9. It is known that Beyoncé wrote the lyrics for all of the Destiny's child songs. The average word length for these songs is 3.64 letters. Based on your samples, do you have good evidence that Beyoncé did not write the lyrics for Crazy in Love. Explain.

Name: _____ Hour: _____ Date: _____

Lesson 4.1 – Random Sampling

Important ideas:

Check Your Understanding

A curious student wants to know the average height for all students in the school. She selects a random sample of 40 students, measures their heights, and calculates the mean height to be 65.2 inches.

1. If she were to take a *different* random sample of 40 students, do you think she would get exactly the same result? Explain.

2. Do you think her sample mean of 65.2 inches is exactly the same as the true mean height for all students at the school? Why?

3. Would it be reasonable for the student to make each of the following claims? Explain.

   a. *I am confident that the mean height for the* ***sample of students*** *is 65.2 inches.*

   b. *I am confident that the mean height of* ***students at my school*** *is close to 65.2 inches.*

   c. *I am confident that the mean height of* ***all high school students*** *is close to 65.2 inches.*